NASA TT F-8058
AUG 20 1961

[redacted]

DRAFT TRANSLATION

DYNAMICS OF VISCOELASTIC SHELLS AND PLATES

(Dinamika uprugo-vyazkikh obolochek i plastin)

Doklady A N SSSR
T. 138, No.6, 196[redacted]
pp. 1317-1320.

by E. I. Grigolyuk
Member-correspondent
of the USSR Ac. of Sc.

The general theory of oscillations of viscoelastic shells is described in the present paper. The material of the shell is considered isotropic, homogenous, and dependent upon the linear relationship between three tensors — stresses, stress velocity and deformation velocity. Lyava-Kirchhoff hypotheses are valid for the shell. The shell is considered sloping, the shifting of its surface's median part being small. A system of differential equations of the problem is obtained. It is solved in the case of a circular cylindrical shell flown around by a supersonic gas stream along its genetratrix.

1. Visoelastic sloping shells. The equilibrium equations of the shell's small element have the form

$$\frac{\partial N_1}{\partial x} + \frac{\partial T_2}{\partial y} + X = 0, \quad \frac{\partial N_2}{\partial y} + \frac{\partial T_1}{\partial x} + Y = 0, \quad \frac{N_1}{R_1} + \frac{N_2}{R_2} + \frac{\partial Q_1}{\partial x} + \frac{\partial Q_2}{\partial y} + Z = 0,$$
$$\frac{\partial H_1}{\partial x} + \frac{\partial M_2}{\partial y} - Q_2 = 0, \quad \frac{\partial H_2}{\partial y} + \frac{\partial M_1}{\partial x} - Q_1 = 0; \qquad (1,1)$$

Here $N_1, N_2, T_1 = T_2 = T, \ Q_1, Q_2, \ M_1, \ M_2, H_1 = H_2 = H$ are the specific forces and moments; X, Y, Z are the components of the external surface stresses, respectively along the orthogonal axes x, y, z. R_1, R_2 are the main curvature radiuses. The system of coordinates coincides with the main directions on the median surface.

If $\sigma_1(z), \ \sigma_2(z), \ \tau_{12}(z)$ are the stresses, we have

$$N_1 = \int_{-h/2}^{h/2} \sigma_1(z)\,dz, \quad N_2 = \int_{-h/2}^{h/2} \sigma_2(z)\,dz, \quad T = \int_{-h/2}^{h/2} \tau_{12}(z)\,dz,$$
$$M_1 = \int_{-h/2}^{h/2} \sigma_1(z)\,z\,dz, \quad M_2 = \int_{-h/2}^{h/2} \sigma_2(z)\,z\,dz, \quad H = \int_{-h/2}^{h/2} \tau_{12}(z)\,dz. \qquad (1,2)$$

For a linear homogenous and isotropic Maxwell medium at a plane state of stress we have

$$\begin{aligned}
\dot{\varepsilon}_1(z) &= \frac{D_0}{2\mu}\sigma_1(z) + \frac{1}{3}\left(\frac{D_1}{3K_v} - \frac{D_0}{2\mu}\right)(\sigma_1(z) + \sigma_2(z)) = \\
&= \frac{1}{3}\left(\frac{D_0}{\mu} + \frac{D_1}{3K_v}\right)\sigma_1(z) - \frac{1}{3}\left(\frac{D_0}{2\mu} - \frac{D_1}{3K_v}\right)\sigma_2(z), \\
\dot{\varepsilon}_2(z) &= \frac{D_0}{2\mu}\sigma_2(z) + \frac{1}{3}\left(\frac{D_1}{3K_v} - \frac{D_0}{2\mu}\right)(\sigma_1(z) + \sigma_2(z)) = \\
&= \frac{1}{3}\left(\frac{D_0}{\mu} + \frac{D_1}{3K_v}\right)\sigma_2(z) - \frac{1}{3}\left(\frac{D_0}{2\mu} - \frac{D_1}{3K_v}\right)\sigma_1(z), \quad \dot{\gamma}_{12}(z) = \frac{D_0}{\mu}\tau_{12}(z),
\end{aligned} \tag{1,3}$$

where $\varepsilon_1(z)$, $\varepsilon_2(z)$ are the relative deformations of the surface z = const along the axes x and y; $\gamma_{12}(z)$ is the antle of surface's shift (z = con.) $K_v = {}^2/_3\mu + \lambda$ is the volume viscosity; μ, λ are the coefficients of hardness: G is the shift module; $K_c = 2G(1+\nu)/3(1-2\nu)$; ν is the Poisson coefficient. The point designates the differentiation along the time t. At the same time

$$D_0 = 1 + \frac{\mu}{G}\frac{\partial}{\partial t}, \quad D_1 = 1 + \frac{K_v}{K_c}\frac{\partial}{\partial t}. \tag{1,4}$$

According to (1.3), the stresses are

$$\begin{aligned}
\left(\frac{D_0}{2\mu} + \frac{2}{3}\frac{D_1}{K_v}\right)D_0\sigma_1(z) &= 2\mu\left[\left(\frac{D_0}{\mu} + \frac{D_1}{3K_v}\right)\dot{\varepsilon}_1(z) + \left(\frac{D_0}{2\mu} - \frac{D_1}{3K_v}\right)\dot{\varepsilon}_2(z)\right], \\
\left(\frac{D_0}{2\mu} + \frac{2}{3}\frac{D_1}{K_v}\right)D_0\sigma_2(z) &= 2\mu\left[\left(\frac{D_0}{\mu} + \frac{D_1}{3K_v}\right)\dot{\varepsilon}_2(z) + \left(\frac{D_0}{2\mu} - \frac{D_1}{3K_v}\right)\dot{\varepsilon}_1(z)\right], \\
D_0\tau_{12}(z) &= \dot{\gamma}_{12}(z).
\end{aligned} \tag{1,5}$$

The Kirchhoff- Lyav hypotheses lead for deformations to the expressions :

$$\varepsilon_1(z) = \varepsilon_1 - z\varkappa_1, \quad \varepsilon_2(z) = \varepsilon_2 - z\varkappa_2, \quad \gamma_{12}(z) = \gamma_{12} - 2z\varkappa_{12}, \tag{1,6}$$

whereupon the relative deformations ε_1 and ε_2, and the shift angle γ_{12} of the median surface for a sloping shell at small shifts have the form

$$\begin{aligned}
\varepsilon_1 &= \frac{\partial u}{\partial x} - \frac{w}{R_1}, \quad \varepsilon_2 = \frac{\partial v}{\partial y} - \frac{w}{R_2}, \quad \gamma_{12} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}, \\
\varkappa_1 &= \frac{\partial^2 w}{\partial x^2}, \quad \varkappa_2 = \frac{\partial^2 w}{\partial y^2}, \quad \varkappa_{12} = \frac{\partial^2 w}{\partial x\,\partial y}.
\end{aligned} \tag{1,7}$$

Introducing (1.5) into (1.2), and bearing in mind (1.6), we shall obtain

...

$$\left(\frac{D_0}{2\mu}+\frac{2}{3}\frac{D_1}{K_v}\right)D_0M_1=-\frac{\mu h^3}{6}\left[\left(\frac{D_0}{\mu}+\frac{D_1}{3K_v}\right)\dot{\varkappa}_1+\left(\frac{D_0}{2\mu}-\frac{D_1}{3K_v}\right)\dot{\varkappa}_2\right],$$
$$\left(\frac{D_0}{2\mu}+\frac{2}{3}\frac{D_1}{K_v}\right)D_0M_2=-\frac{\mu h^3}{6}\left[\left(\frac{D_0}{\mu}+\frac{D_1}{3K_v}\right)\dot{\varkappa}_2+\left(\frac{D_0}{2\mu}-\frac{D_1}{3K_v}\right)\dot{\varkappa}_1\right],$$
$$D_0H=-\frac{\mu h^3}{6}\dot{\varkappa}_{12},\quad D_0T=\mu h\dot{\gamma}_{12},\tag{1,8}$$
$$\left(\frac{D_0}{2\mu}+\frac{2}{3}\frac{D_1}{K_v}\right)D_0N_1=2\mu h\left[\left(\frac{D_0}{\mu}+\frac{D_1}{3K_v}\right)\dot{\varepsilon}_1+\left(\frac{D_0}{2\mu}-\frac{D_1}{3K_v}\right)\dot{\varepsilon}_2\right],$$
$$\left(\frac{D_0}{2\mu}+\frac{2}{3}\frac{D_1}{K_v}\right)D_0N_2=2\mu h\left[\left(\frac{D_0}{\mu}+\frac{D_1}{3K_v}\right)\dot{\varepsilon}_2+\left(\frac{D_0}{2\mu}-\frac{D_1}{3K_v}\right)\dot{\varepsilon}_1\right].$$

Now, introducing (1.6) into (1.5), multiplying both terms of the expression by dz, and then by zdz, and integrating the result along the thickness of the shell, we find

$$\dot{\varepsilon}_1=\frac{1}{3h}\left[\left(\frac{D_0}{\mu}+\frac{D_1}{3K_v}\right)N_1-\left(\frac{D_0}{2\mu}-\frac{D_1}{3K_v}\right)N_2\right],$$
$$\dot{\varepsilon}_2=\frac{1}{3h}\left[\left(\frac{D_0}{\mu}+\frac{D_1}{3K_v}\right)N_2-\left(\frac{D_0}{2\mu}-\frac{D_1}{3K_v}\right)N_1\right],$$
$$\dot{\gamma}_{12}=\frac{1}{\mu h}D_0T,\quad \dot{\varkappa}_{12}=-\frac{6}{\mu h^3}D_0H,\tag{1,9}$$
$$\dot{\varkappa}_1=-\frac{4}{h^3}\left[\left(\frac{D_0}{\mu}+\frac{D_1}{3K_v}\right)M_1-\left(\frac{D_0}{2\mu}-\frac{D_1}{3K_v}\right)M_2\right],$$
$$\dot{\varkappa}_2=-\frac{4}{h^3}\left[\left(\frac{D_0}{\mu}+\frac{D_1}{3K_v}\right)M_2-\left(\frac{D_0}{2\mu}-\frac{D_1}{3K_v}\right)M_1\right].$$

From the first three relations (1.7), we obtain the deformation consistency equation

$$\frac{\partial^2\varepsilon_1}{\partial y^2}+\frac{\partial^2\varepsilon_2}{\partial x^2}-\frac{\partial^2\gamma_{12}}{\partial x\,\partial y}=-\frac{1}{R_1}\frac{\partial^2 w}{\partial y^2}-\frac{1}{R_2}\frac{\partial^2 w}{\partial x^2},\tag{1,10}$$

and upon integrating in time, we obtain the deformation velocity consistency equation

$$\frac{\partial^2\dot{\varepsilon}_1}{\partial y^2}+\frac{\partial^2\dot{\varepsilon}_2}{\partial x^2}-\frac{\partial^2\dot{\gamma}_{12}}{\partial x\,\partial y}=-\frac{1}{R_1}\frac{\partial^2\dot{w}}{\partial y^2}-\frac{1}{R_2}\frac{\partial^2\dot{w}}{\partial x^2}.\tag{1,11}$$

The first two equilibrium equations (1.1) are satisfied by the force function F, at X = Y = 0, determined as

$$N_1=\frac{\partial^2 F}{\partial y^2},\quad N_2=\frac{\partial^2 F}{\partial x^2},\quad T=-\frac{\partial^2 F}{\partial x\,\partial y}.\tag{1,12}$$

We now introduce the fourth and fifth equations (1.1) into the third, and substitute for M_1, M_2 and H their values according to relations (1.8) and for N_1 and N_2 their expressions through the force function (1.12). The first three dependences (1.9) are then introduced into (1.11), taking into account (1.12). As a result, we obtain

$$\left(\frac{D_0}{\mu}+\frac{D_1}{3K_v}\right)\nabla^4\dot{w}-\frac{6}{h^3}\left(\frac{D_0}{2\mu}+\frac{2}{3}\frac{D_1}{K_v}\right)\frac{D_0}{\mu}\left[\frac{1}{R_1}\frac{\partial^2F}{\partial y^2}+\frac{1}{R_2}\frac{\partial^2F}{\partial x^2}+Z\right]=0, \quad (1,13)$$

$$\left(\frac{D_0}{\mu}+\frac{D_1}{3K_v}\right)\nabla^4F=-3h\left(\frac{1}{R_1}\frac{\partial^2\dot{w}}{\partial y^2}+\frac{1}{R_2}\frac{\partial^2\dot{w}}{\partial x^2}\right) \quad (1,14)$$

$$\left(\nabla^4=\frac{\partial^4}{\partial x^4}+2\frac{\partial^4}{\partial x^2\,\partial y^2}+\frac{\partial^4}{\partial y^4}\right).$$

It must be understood that Z of (1.13) represents the transverse load introduced, which in case of a shell with the initial conditions and the action of the external medium is

$$Z=N_1^0\frac{\partial^2w}{\partial x^2}+N_2^0\frac{\partial^2w}{\partial y^2}+2T^0\frac{\partial^2w}{\partial x\,\partial y}+\frac{\gamma h}{g}\ddot{w}+p_c, \quad (1,15)$$

where γ is the specific weight of the material of the shell ; g - gravitation acceleration ; p_c is the pressure of the medium on the surface of the shell.

From (1.13) — (1.14) it is possible to obtain a single differential equation related to the deflection

$$\left(\frac{D_0}{\mu}+\frac{D_1}{3K_v}\right)^2\nabla^8\dot{w}+\frac{6}{h^3}\left(\frac{D_0}{2\mu}+\frac{2}{3}\frac{D_1}{K_v}\right)\frac{D_0}{\mu}\left[3h\left(\frac{1}{R_1^2}\frac{\partial^4\dot{w}}{\partial x^4}+\frac{2}{R_1R_2}\frac{\partial^4\dot{w}}{\partial x^2\,\partial y^2}+\right.\right.$$
$$\left.+\frac{1}{R_2^2}\frac{\partial^4\dot{w}}{\partial x^4}\right)-\left(\frac{D_0}{\mu}+\frac{D_1}{3K_v}\right)\nabla^4\left(N_1^0\frac{\partial^2w}{\partial x^2}+N_2^0\frac{\partial^2w}{\partial y^2}+\right.$$
$$\left.\left.+2T^0\frac{\partial^2w}{\partial x\,\partial y}+\frac{\gamma h}{g}\ddot{w}+p_c\right)\right]=0. \quad (1,16)$$

If we now introduce a new function Φ by means of

$$F=3h\left(\frac{1}{R_1}\frac{\partial^2\Phi}{\partial y^2}+\frac{1}{R_2}\frac{\partial^2\Phi}{\partial x^2}\right),\quad \dot{w}=-\left(\frac{D_0}{\mu}+\frac{D_1}{3K_v}\right)\nabla^4\Phi, \quad (1,17)$$

the equation (1.14) is identically satisfied, and (1.13) will take the form

$$\left(\frac{D_0}{\mu}+\frac{D_1}{3K_v}\right)^2\nabla^8\Phi+\frac{18}{h^2}\left(\frac{D_0}{2\mu}+\frac{2}{3}\frac{D_1}{K_v}\right)\frac{D_0}{\mu}\left\{\frac{1}{R_1^2}\frac{\partial^4\Phi}{\partial y^4}+\frac{2}{R_1R_2}\frac{\partial^4\Phi}{\partial x^2\,\partial y^2}+\frac{1}{R_2^2}\frac{\partial^4\Phi}{\partial x^4}+\right.$$
$$\left.+\frac{1}{3h}\left[N_1^0\frac{\partial^2w}{\partial x^2}+N_2^0\frac{\partial^2w}{\partial y^2}+2T^0\frac{\partial^2w}{\partial x\,\partial y}-\frac{\gamma h}{g}\left(\frac{D_0}{\mu}+\frac{D_1}{3K_v}\right)\nabla^4\Phi+p_c\right]\right\}=0.$$

2. Oscillations of a Cylindrical Shell in Supersonic Gas Flow.

Let a cylindrical shell be submitted to an internal transverse pressure p, while being flown about by a supersonic gas stream from the outside. We then shall have

$$R_1=\infty,\quad R_2=R,\quad y=R\varphi,\quad T^0=0,\quad N_1^0={}^1/_2\,pR,\quad N_2^0=pR, \tag{2.1}$$
$$-q=q_1\frac{\partial w}{\partial t}+q_2\frac{\partial w}{\partial x}\quad\left(q_1=\frac{\rho U}{\sqrt{M^2-1}},\quad q_2=\frac{\rho U^2}{\sqrt{M^2-1}}\right).$$

Here q is the complementary pressure in the gas flow on account of the deflection of the shell from the unperturbed cylindrical form at oscillations, and it corresponds to the theory of a stationary supersonic flow; U is the velocity of the unperturbed flow; M = U/c ; c is the velocity of sound propagation in the unpeturbed flow ; ρ is the density of the flow.

Let us introduce (2.1) into the equations (1.13 – (1.14), and let us search for the solution of these equations in the form of traveling waves

$$w=w_0e^{i(\omega t-kx)}\cos n\varphi,\qquad F=F_0e^{i(\omega t-kx)}\cos n\varphi, \tag{2.2}$$

where $_0$, F, k are constants ; n is the number of half-waves in a circular direction ; ω is the angular frequency of the oscillations of the shell in the flow.

The substitution of (2.2) into (1.13) – (1.14) leads to the following equation for the introduced frequency ω^* of oscillations of the shell in the flow :

$$\omega^{*5}-i\omega^{*4}a_4-\omega^{*3}(a_{32}-ia_{31})-\omega^{*2}(a_{22}+ia_{21})+\omega^*(a_{12}-ia_{11})+$$
$$+a_{02}+ia_{01}=0. \tag{2.3}$$

In addition to this

$$a_4=\frac{1}{3(1-\nu^2)\pi}\left(\frac{3-\nu-\nu^2}{\mu^*}+\frac{3+2\nu-\nu^2}{3K_\nu^*}\right)+\frac{q_1^*}{\pi},\qquad a_{31}=\frac{q_2^*hk}{\pi^2},$$

$$a_{32}=\frac{\theta}{\pi^2}\left[\frac{h^2}{R^2}\frac{k^4}{(k^2+n^2/R^2)^2}+\frac{h^4}{12(1-\nu^2)}\left(k^2+\frac{n^2}{R^2}\right)^2\right]+N_1^*\frac{k^2R^2}{\pi^2}+N_2^*\frac{n^2}{\pi^2}+$$
$$+\frac{1}{18(1-\nu^2)\pi^2}\left(\frac{11-4\nu}{2\mu^{*2}}+2\frac{7+\nu}{3K_\nu^*\mu^*}+\frac{4}{9}\frac{1+\nu}{K_\nu^{*2}}\right)+$$
$$+\frac{1}{3(1-\nu^2)}\frac{q_1^*}{\pi^2}\left(\frac{3-\nu-\nu^2}{\mu^*}+\frac{3+2\nu-\nu^2}{3K_\nu^*}\right),$$

$$a_{22} = \frac{q_2^* hk}{3(1-\nu^2)\pi^3}\left(\frac{3-\nu-\nu^2}{\mu^*} + \frac{3+2\nu-\nu^2}{3K_\nu^*}\right),$$

$$a_{21} = \frac{1}{3(1-\nu^2)\pi^3}\left\{\theta\left[\frac{h^2}{R^2}\frac{k^4}{(k^2+n^2/R^2)^2}\left(\frac{2-\nu}{\mu^*} + 2\frac{1+\nu}{3K_\nu^*}\right) + \right.\right.$$
$$+ \left.\frac{1}{6}h^4\left(k^2+\frac{n^2}{R^2}\right)^2\left(\frac{1}{\mu^*}+\frac{1}{3K_\nu^*}\right)\right] + \left(\frac{3-\nu-\nu^2}{\mu^*} + \frac{3+2\nu-\nu^2}{3K_\nu^*}\right)(N_1^* R^2 k^2 +$$
$$\left. + N_2^* n^2) + \frac{1}{6}\frac{1}{\mu^*}\left(\frac{1}{2\mu^*}+\frac{2}{3K_\nu^*}\right)\left(\frac{1}{\mu^*}+\frac{1}{3K_\nu^*}\right) + \frac{1}{6}q_1^*\left(\frac{11-4\nu}{2\mu^{*2}} + 2\frac{7+\nu}{3K_\nu^*\mu^*} + \frac{4}{9}\frac{1+\nu}{K_\nu^{*2}}\right)\right\},$$

$$a_{12} = \frac{1}{6(1-\nu^2)\pi^4}\left\{\theta\left[\frac{h^2}{R^2}\frac{k^4}{(k^2+n^2/R^2)^2}\frac{1}{\mu^*}\left(\frac{1}{2\mu^*}+\frac{2}{3K_\nu^*}\right) + \right.\right.$$
$$+ \left.\frac{1}{48(1-\nu^2)}h^4\left(k^2+\frac{n^2}{R^2}\right)^2\left(\frac{1}{\mu^*}+\frac{1}{3K_\nu^*}\right)^2\right] +$$
$$+ \frac{1}{3}\left(\frac{11-7\nu}{2\mu^{*2}} + 2\frac{7+\nu}{3K^*\mu^*} + \frac{4}{9}\frac{1+\nu}{K_\nu^{*2}}\right)(N_1^* R^2 k^2 + N_2^* n^2) +$$
$$\left. + \frac{1}{3}\frac{q_1^*}{\mu^*}\left(\frac{1}{2\mu^*}+\frac{2}{3K_\nu^*}\right)\left(\frac{1}{\mu^*}+\frac{1}{3K_\nu^*}\right)\right\},$$

$$a_{11} = \frac{q_2^* hk}{18(1-\nu^2)\pi^4}\left(\frac{11-4\nu}{2\mu^{*2}} + 2\frac{7+\nu}{3K_\nu^*\mu^*} + \frac{4}{9}\frac{1+\nu}{K_\nu^{*2}}\right),$$

$$a_{02} = \frac{q_2^* hk}{18(1-\nu^2)\pi^5}\frac{1}{\mu^*}\left(\frac{1}{2\mu^*}+\frac{2}{3K_\nu^*}\right)\left(\frac{1}{\mu^*}+\frac{1}{3K_\nu^*}\right),$$

$$a_{01} = \frac{1}{18(1-\nu^2)\pi^5}\frac{1}{\mu^*}\left(\frac{1}{2\mu^*}+\frac{2}{3K_\nu^*}\right)\left(\frac{1}{\mu^*}+\frac{1}{3K_\nu^*}\right)(N_1^* R^2 k^2 + N_2^* n^2).$$

$$\omega^* = \frac{\omega h}{\pi c}, \quad \theta = \frac{gE}{\gamma c^2}, \quad \mu^* = \frac{\mu c}{Eh}, \quad K_\nu^* = \frac{K_\nu c}{Eh}, \quad q_1^* = \frac{q_1 g}{\gamma c} = \frac{\rho g}{\gamma}\frac{M}{\sqrt{M^2-1}},$$
$$q_2^* = \frac{q_2 g}{\gamma c^2} = \frac{\rho g}{\gamma}\frac{M^2}{\sqrt{M^2-1}}, \quad N_1^* = \frac{gN_1^0 h}{\gamma c^2 R^2}, \quad N_2^* = \frac{gN_2^0 h}{\gamma c^2 R^2}.$$

The frequency introduced in (2.3) is a complex magnitude.

*** THE END ***

Institute of Hydrodynamics of the
Siberian Division
of the USSR Academy of Sciences

Entered on
27 January 1961.

Translated by ANDRE L. BRICHANT
at NASA TECHNICAL INFORMATION DIVISION
on 20 August 1961.

../..

REFERENCES.

1. C. TORRE., Österreich. Ing. Arch., 8, H, 1, 55 (1954)

2. C. TORRE., Koll. Zs., 138, H.1, 11 (1954)

3. J. MARTINEK, G. C. K. YEH, Actes du IXe Congres International de Mécanique Appliquée. T. 5., Bruxelles, 1957, pp. 340-351, 352-359.

4. V. Z. VLASOV. Obshchaya teoriya obolochek iyeye prilozheniya v tekhnike (General Theory of Shells and its Application in Technology). Moscow-Leningrad, State Publishing House for Technical-Theoretical Literature, 1949. 784 pp. NASA Translation in process.

5. A. FERRI., Aerodinamika sverkhzvukovykh techeniy, (Aerodynamics of Supersonic Flows), 1952.